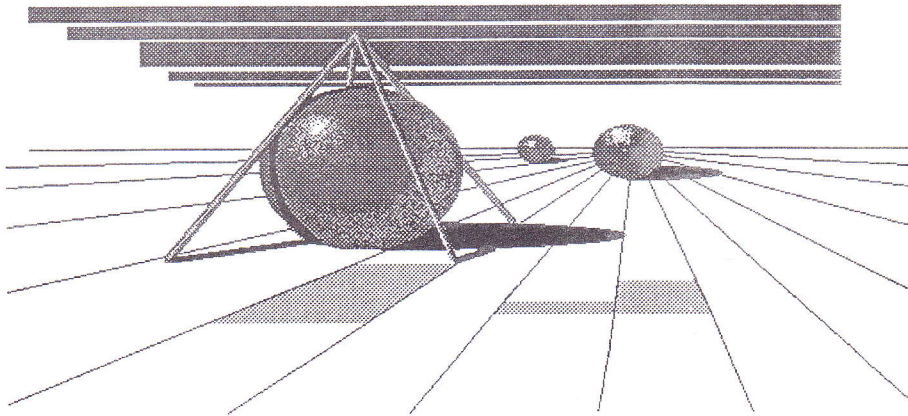


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## EFFICIENT ECONOMIC LOAD DISPATCH USING HOPFIELD MODELING

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### ABSTRACT

The solution of the economic load dispatch problem using Hopfield modeling is presented. It has been intended to include the power limits constraints of the generating units into the energy function in addition to the total fuel cost, transmission losses and the power balance constraint. The weighting factors of the energy function are appropriately selected, while the weighting factor associated with the transmission losses may be adjusted and the generated power of each unit in addition to the transmission losses may be updated during the computation process. The proposed method is applied on a power system. The obtained results are compared with the results which are obtained by application of some conventional methods on the power system such as LaGrange method, second order gradient method and participation factors method. As a recent method, the artificial neural network is also designated to solve the economic dispatch problem. The acceptable agreement between the obtained results reveals the validity and verifies the feasibility of the proposed method. The solution could be obtained without violation of the power limits, consequently, without compulsion of the generating units to dispatch the load on these limits.

ملخص باللغة العربية :

أسلوب فعال للتوزيع الاقتصادي للأحمال باستخدام نموذج هوبفيلد

تم تقديم طريقة مقترحة لتوزيع الأحمال على وحدات التوليد بأقل تكلفة ممكنة وذلك باستخدام نموذج هوبفيلد وتم اقتراح إدخال ضوابط نهايات القدرة لهذه الوحدات ضمن دالة الطاقة وذلك بالإضافة إلى التكلفة الكلية، القدرة المفقودة، ومعادلة إتران القدرة. ويتم إختيار معاملات المعايير لدالة الطاقة مع

ضبط معامل معايرة القدرة المفقودة ويتم أثناء الحسابات التحديث لقيم القدرة المولدة من الوحدات وكذلك القدرة المفقودة. وتم عمل نموذج رياضي للطريقة المقترحة ، والتطبيق علي نظام قوي في حالتي عدم الأخذ في الإعتبار ، وإعتبار القدرة المفقودة ، ومقارنة النتائج التي تم الحصول عليها مع نتائج تطبيق بعض الطرق التقليدية مثل طريقة لاجرانج ، وطريقة التدرج للمعامل الثاني ، وطريقة معاملات المشاركة. كما تم تطبيق الشبكات العصبية الإصطناعية كطريقة حديثة للتطبيق وتم تصميم الشبكة من حيث عدد المدخلات والمخرجات ومعاملات التدريب وثوابت العزم. وأثبت التوافق المقبول للنتائج صلاحية الطريقة المقترحة وإمكانية إجراؤها ، خاصة وأن نتائجها تحقق ضوابط نهايات القدرة لكل وحدة توليد دون أن تتعداها، ودون إجبار لهذه القدرات لتكون علي الحدود الصغري أو الحدود العليا.

## KEYWORDS

Hopfield method, Economic load dispatch, Economic operation, Energy function, Artificial Neural Network.

## 1. INTRODUCTION

With the development of integrated power systems and the interconnection of operating companies for purposes of economy interchange, it is necessary to consider not only the incremental fuel costs but also the incremental transmission losses for optimal economic operation. Much effort has been expand in the past to solve the economic load dispatch of power systems [1, 2, and 3].

The objective function of a power system, which is connected to an equivalent load bus through a transmission network, is to minimize the total fuel cost (KT).

$$KT = \sum_{i=1}^N K_i(P_i) \quad (1)$$

Where  $K_i(P_i)$  is the cost equation of each unit, it can be given in terms of the characteristic constants  $a_i$ ,  $b_i$  and  $c_i$  of each unit.

$$K_i(P_i) = a_i + b_i P_i + c_i P_i^2 \quad (2)$$

The power balance constraint states that, the sum of the generated power of each unit ( $P_i$ ) must equal to the received load (PR) plus the transmission losses (PL).



$$\sum_{i=1}^N P_i = PR + PL \quad (3)$$

The power limits constraint defines that, the power output of each unit must be less than or equal to the maximum power permitted ( $PX_i$ ) and must be greater than or equal to the minimum power permitted on that unit ( $PN_i$ ).

$$PN_i \leq P_i \leq PX_i \quad (4)$$

The inequality constraints can be transformed to equality constraints by the following equation.

$$(PX_i - P_i)(P_i - PN_i) + \eta_i = 0 \quad (5)$$

Where  $\eta_i$  is a control variable for the generating unit  $i$ , its value changes during the optimization process and from time interval (IT) to other one. It equals zero when  $P_i$  equals  $PX_i$  or  $PN_i$  and it has a negative value for the values of  $P_i$  which are between the two power limits.

According to LaGrange method [1], get starting values for the generated power of each unit and pick a starting value of the incremental cost of received power  $\lambda$ , then calculate PL from the transmission loss formula. The economic load dispatch problem has to be solved repeatedly until the following power balance constraint would be satisfied.

$$PR + PL - \sum_{i=1}^N P_i \leq \epsilon \quad (6)$$

Where  $\epsilon$  is the total demand tolerance, which is taken as  $10^{-6}$  (also for all other applied methods).

Gradient search techniques always start off with a feasible solution, in which all constraints are satisfied, and search for the optimum solution until no significant gain in the objective function is obtained [1, 2]. By using the participation factors method, consistent answers and a straight forward solution can be obtained repeatedly by moving the generators from one economically optimum schedule to another as the load changes by a reasonably small amounts [1].

When the obtained solution by one of these conventional methods gives for a certain generating unit a value of the generated power, which violates the

corresponding power limits, the solution may be modified with compulsion this unit to operate on that violated power limit.

Recently, the economic load dispatch problem has been solved by using artificial neural networks [4, 5] and by using Hopfield modeling [6, 7, 8, 13]. The mathematical model for the proposed method to solve the problem based on Hopfield modeling is presented. It is intended to extend the energy function of Hopfield modeling to contain the inequality constraints of the power limits after their transformation to equality constraints to avoid the necessity to observe the power limits during the course of the computation. Also, to avoid the compulsion of the generating units to operate without violation of these limits. Comparisons are carried out between the results, which are obtained by the all applied methods.

## 2. Mathematical modeling of the proposed method

The energy function (EF) of the Hopfield model, which may be converged during the computation process [7, 8, 13], can be defined by

$$EF = -0.5 \sum_i \sum_j Z_{ij} y_i y_j - \sum_i O_i y_i \quad (7)$$

Where  $y_i$ ,  $y_j$  are the outputs of neuron  $i$  and neuron  $j$ , respectively,  $Z_{ij}$  is the mutual conductance between neuron  $i$  and neuron  $j$ , and  $O_i$  is the external input to neuron  $i$ . The relation between the input and the output for each neuron is given by the following dynamic characteristic

$$\dot{I}_i = \sum_j Z_{ij} y_j + O_i \quad (8)$$

Where  $I_i$  is the input to the neuron  $i$ ,  $\dot{I}_i$  is the rate of change of  $I_i$  with respect to the time.

To apply the Hopfield model on the economic load dispatch problem, the energy function (EF) is extended to include the objective function, Eqs. (1) and (2), power balance constraint, Eq. (3), and the transmission losses (PL) in addition to the equality constraints of power limits of each unit, Eq. (5).

$$EF = 0.5 \alpha [PR + PL - \sum_i P_i]^2 + 0.5 \beta \sum_i (a_i + b_i P_i + c_i P_i^2) + 0.5 \gamma PL + 0.5 \delta \sum_i [(PX_i - P_i) (P_i - PN_i) + \eta_i] \quad (9)$$

Where  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$  are weighting factors. The transmission losses (PL) and the penalty factor (PF<sub>i</sub>) of each unit are given, respectively, by

$$PL = \sum_{i=1}^N \sum_{j=1}^N P_i B_{ij} P_j \quad (10)$$

$$PF_i = 1 / (1 - \delta PL / \delta P_i) = 1 / (1 - LI_i') \quad (11)$$

Where  $B_{ij}$  are the coefficient of the transmission losses formula,  $LI_i'$  is the incremental loss of unit  $i$  at an initial generation  $P_i'$ .

$$LI_i' = \delta PL / \delta P_i' = 2 \sum_j B_{ij} P_j \quad (12)$$

When the generated power of unit  $i$  changes from  $P_i'$  to  $P_i$ , the transmission losses will be changed from  $PL'$  to  $PL$ .

$$PL = PL' + \Delta PL = PL' + \sum_i LI_i' (P_i - P_i') \quad (13)$$

Substituting Eq. (13) into Eq. (9), the energy function (EF) becomes

$$\begin{aligned} EF = & [0.5 \alpha (PR + PL')^2 + 0.5 \beta \sum_i a_i + 0.5 \gamma (PL' - \sum_i LI_i' P_i') - 0.5 \delta \sum_i PX_i PN_i \\ & + 0.5 \delta \sum_i \eta_i] - \sum_i [\alpha (PR + PL') - 0.5 \beta b_i - 0.5 \gamma LI_i' - \delta AP_i] P_i \\ & + \sum_i 0.5 (\alpha + \beta c_i - \delta) P_i^2 + \sum_i \sum_j 0.5 \alpha P_i P_j \end{aligned} \quad (14)$$

Where  $AP_i$  is the average generation of unit  $i = 0.5 (PX_i + PN_i)$

The first term of Eq. (14) is constant. Comparing Eqs. (7) and (14) yields

$$Z_{ii} = -\alpha - \beta c_i + \delta \quad (15)$$

$$Z_{ij} = -\alpha \quad (16)$$

$$O_i = \alpha (PR + PL') - 0.5 \beta b_i - 0.5 \gamma LI_i' - \delta AP_i \quad (17)$$

Substituting Eqs. (15)-(17) into Eq. (8) gives

$$\dot{I}_i = \alpha Y - 0.5 \beta (b_i + 2 c_i P_i) - 0.5 \gamma LI_i' + \delta P_i - \delta AP_i \quad (18)$$

Where,

$$Y = PR + PL' - \sum_j P_j \quad (19)$$

The generated power of each unit  $P_i$  is a function of the input to neuron  $i$ ,  $f_i(I_i)$ , [6, 7, 8, 13],

$$P_i = f_i(I_i) = - \frac{(IX_i - I_i)}{(IX_i - IN_i)} (PX_i - PN_i) + PX_i \quad (20)$$

Where  $IX_i$ ,  $IN_i$  are the maximum and minimum limit of  $I_i$ , respectively. Then,

$$P_i' = \frac{(PX_i - PN_i)}{(IX_i - IN_i)} I_i' = k I_i' \quad (21)$$

Where  $k$  is a function of maximum and minimum limits of both generated power of unit  $i$  and input to neuron  $i$ . Substituting Eq. (18) into Eq. (21) and solving in the optimal generation  $P_i$  of unit  $i$  yields

$$P_i = \frac{\alpha Y - M_i}{\beta c_i - \delta} \quad (22)$$

Where,

$$M_i = 0.5 \beta b_i + \gamma \sum_j B_{ij} P_j + \delta A P_i \quad (23)$$

From Eqs.(19), (22) and (23),  $Y$  will be given by

$$Y = \frac{PR + PL' + \sum_i^N [M_i / (\beta c_i - \delta)]}{1 + \sum_i^N [\alpha / (\beta c_i - \delta)]} \quad (24)$$

The incremental fuel cost of unit  $i$  is given by

$$d F_i / d P_i = b_i + 2 c_i P_i \quad (25)$$

From Eqs.(21) and (18), the incremental fuel cost can be obtained in terms of the weighting factors  $\alpha$ ,  $\beta$  and  $\gamma$

$$P_i = K [\alpha Y - 0.5 \beta (b_i + 2 c_i P_i) - 0.5 \gamma L I_i'] = 0 \quad (26)$$



Then,

$$d F_i / d P_i = 2 [\alpha Y - 0.5 \gamma L_i'] / \beta \quad (27)$$

The multiplication of the equal incremental cost and the penalty factor of each unit may be satisfied and the equal constant value ( $\lambda$ ).

$$P F_i (d F_i / d P_i) = 2 \alpha Y / \beta = \lambda \quad (28)$$

Then, the weighting factor of the transmission losses  $\gamma$  may be equal twice the multiplication of  $\alpha$  and  $Y$  as in the following equation.

$$\gamma = 2 \alpha Y \quad (29)$$

The procedure of the calculations of the proposed method is illustrated in the flow chart given in Fig. 1.

### 3. TEST EXAMPLE AND RESULTS

The power system data is given in Table 1. and the transmission loss coefficients of the power system are given in the following matrix, each value may be multiplied by  $10^{-3}$ .

0.05220	0.00864	-0.00406	0.00623	-0.00595
0.00864	0.06770	0.00902	-0.00741	0.00613
-0.00406	0.00902	0.03830	0.00825	0.00289
0.00623	-0.00741	0.00825	0.02955	-0.00396
-0.00595	0.00613	0.00289	-0.00396	0.04632

The data of the daily load curve is given in Table 2, the received power is in Mw and the length of each time interval equals one hour.

When the transmission losses (PL) are not considered in the economic dispatch, the proposed method is applied, first, without inclusion of the power limits constraint of each unit in the energy function. Second, when the energy function is extended to include these constrains. The weights of the energy function are optional selected by  $\alpha = 800000$ ,  $\beta = 0.20$  and  $\gamma = 0.02$  during the optimization period. Table 3 shows the obtained total fuel cost by solving the economic load dispatch problem using the proposed method, LaGrange method, participation factors method and the second-order gradient method. The percentage excess in fuel cost obtained by each method referred to the obtained fuel cost of LaGrange method.



When the transmission losses have been considered, the total fuel cost and the transmission-loss energy obtained by the proposed method and LaGrange method are given in Table 4. The percentage ratio (TTER) between the energy loss and the energy required to cover the received load in the optimization period are also tabulated in Table 4 for both mentioned methods.

Two artificial neural networks (ANN1 & ANN2) are designated by using the back propagation learning algorithm, [11, 12], as a recent method to solve the economic load dispatch problem. Both networks are trained out from the input and output patterns, which are obtained by solving the economic dispatch problem by LaGrange method.

ANN1 is designated for the condition of neglecting the transmission losses and consists of three layers, the first is the input layer which contains 2 neurons. The input to first neuron is the received load and the input to the second neuron is the difference between the received load and the average of the received loads during the optimization period. The intermediate layer is the hidden layer and consists of 4 neurons to connect the input layer and the output layer, which is the third layer that consists of 6 neurons. The outputs for the first five neurons represent the generated power of each unit, while the output of sixth neuron represents the fuel cost of the generating units to cover the received load in each time interval.

When the transmission losses are considered in the calculations, ANN2 is designated, in which the input layer consists of 3 neurons. The inputs to these neurons represent the received load, the difference between the sum of the upper limits of all units and the received load and the difference between the received load and the sum of the lower limits of the generated power of all units, respectively. It is suggested to take these two differences as input data to the network to increase the number of inputs to avoid the saturation caused by the sigmoidal function and to obtain accurate output patterns, particularly with the large number of outputs. The hidden layer consists of five neurons, while the output layer consists of seven neurons (output power for five generating units, the transmission losses and the fuel cost corresponding to each time interval). The input and output patterns may be initialized between 0 and 1 and the connection weights besides the biases may be assumed before the training process of the network [9, 10, 11, 12].

The two networks test four patterns for the output data. The results are compared in Tables 5 and 6 with the corresponding data obtained by solving the problem using LaGrange method and the proposed method. Also, the values of learning rate and the momentum constant of each network are given in each table. These values may be carefully selected to obtain best results.

#### 4. CONCLUSIONS

A proposed method to solve the economic load dispatch problem is presented. The method is based on the Hopfield modeling. The mathematical model has been built with and without extension of the energy function to include the constraint of power limits in addition to the objective function, power balance and the transmission losses. The results obtained by application of the proposed method on a power system have a great agreement with the results obtained by application of some conventional methods as LaGrange method, participation factors method and the second-order gradient method.

The comparison verifies validity and feasibility of the proposed method. Full agreement is existed between the results obtained by the proposed method, especially, with that obtained by LaGrange method, when the energy function contains, only, the objective function, power balance constraint and the transmission losses.

When the energy function is extended in the proposed method to contain the power limits constraints of the generating units, agreeable results for the fuel cost and the ratio of transmission energy loss to the energy required to cover the received load over the optimization period would be noticed.

Artificial neural networks are designated, as recent method, to solve the economic load dispatch problem. These networks comparing with the results obtained by LaGrange method and the proposed method obtain acceptable output data. Using of the neural networks gives wide range of solutions and is valid for the on-line operation.

#### 5. REFERENCES

1. ALEEN, J.W. and WOLLENBERG, B.F., "Power Generation, Operation and Control", John Wiley and Sons, New York, 1984.
2. SALAMA, M.M., ABDEL MAKSOUUD, S.M., "Economic Dispatch of thermal Generation", Modeling, Measurement & Control, D. AMSE Press, Vol.12, No. 2, 1995, pp. 53-63.
3. M. NOR, KH, ABDEL RASHID, A.H., "Efficient Economic Dispatch Algorithm for thermal Unit Commitment", IEE Proceedings-C, Vol. 138, No.3, May 1991, pp. 213-217.



4. SALAMA, M.M., "Optimal Solution to the Hydro-Steam Dispatch Problem using Artificial Neural Networks", Proceeding of the Seventh International Conference on Energy and Environment, Vol.1, Cairo, Egypt, March 2000, pp. 341-351.
5. SALAMA, M.M., "Computing Technique Using ANNs for optimal Scheduling of Hydro-thermal Power Systems with Inclusion of Pump-Storage Plants", Proceedings of the International Conference on Modeling and Simulation, MS'2000, Part 1, Cairo, Egypt, 11-14 April 2000, pp. 109-124.
6. PARK, J.H., KIM, Y.S., EOM, I.K. AND LEE, K.Y., "Economic Load Dispatch for Piecewise Quadratic Cost Function Using Hopfield Neural Network", IEEE Trans. On Power Systems, Vol.8, No. 3, 1993, pp. 1030-1038.
7. SU, C.T. and CHIOU, G.J., "A FAST Computation Hopfield Method to Economic Dispatch of Power Systems", IEEE Trans. On Power Systems, Vol. 12, No. 4, 1997, pp. 1759-1764.
8. SU, C.T., LIN, C.T., LIAW, J.H., "New Approach with a Hopfield Modeling Framework to Economic Dispatch", Proceedings of the Eighth International- Middle-East Power Systems Conference MEPCON'2001, Vol.1, Cairo, Egypt, December 29-31, 2001, pp. 51-55.
9. HOPFIELD, J.J. & TANK, D.W., "Neural Computation of Decisions in Optimization Problems", Biological Cybernetics, Vol. 52, 1985, pp. 141-152.
10. AMARI, S., "Mathematical Foundations of Neuro-Computing", Proceedings of IEEE, Vol.78, No. 9, September 1990, pp. 1443-1462.
11. ROTH, M.W., "Neural-Network Technology and its Applications", John-Hopkins APL Technical Digest, Vol. 9, No. 3, 1988, pp. 1-12.
12. RYME, D.E., HINTON, G.E., WILLIAMS, R.J., "Learning Neural Representation by Error Propagation Parallel Distributed Processing", MIT Press, Vol. 1, 1986, pp. 318-362.
13. SU, C.T., LIN, C.T., "New Approach with a Hopfield Modeling Framework to Economic Dispatch", IEEE Trans. Pwrs, Vol.15, No. 2, May 2000, pp. 541-545.

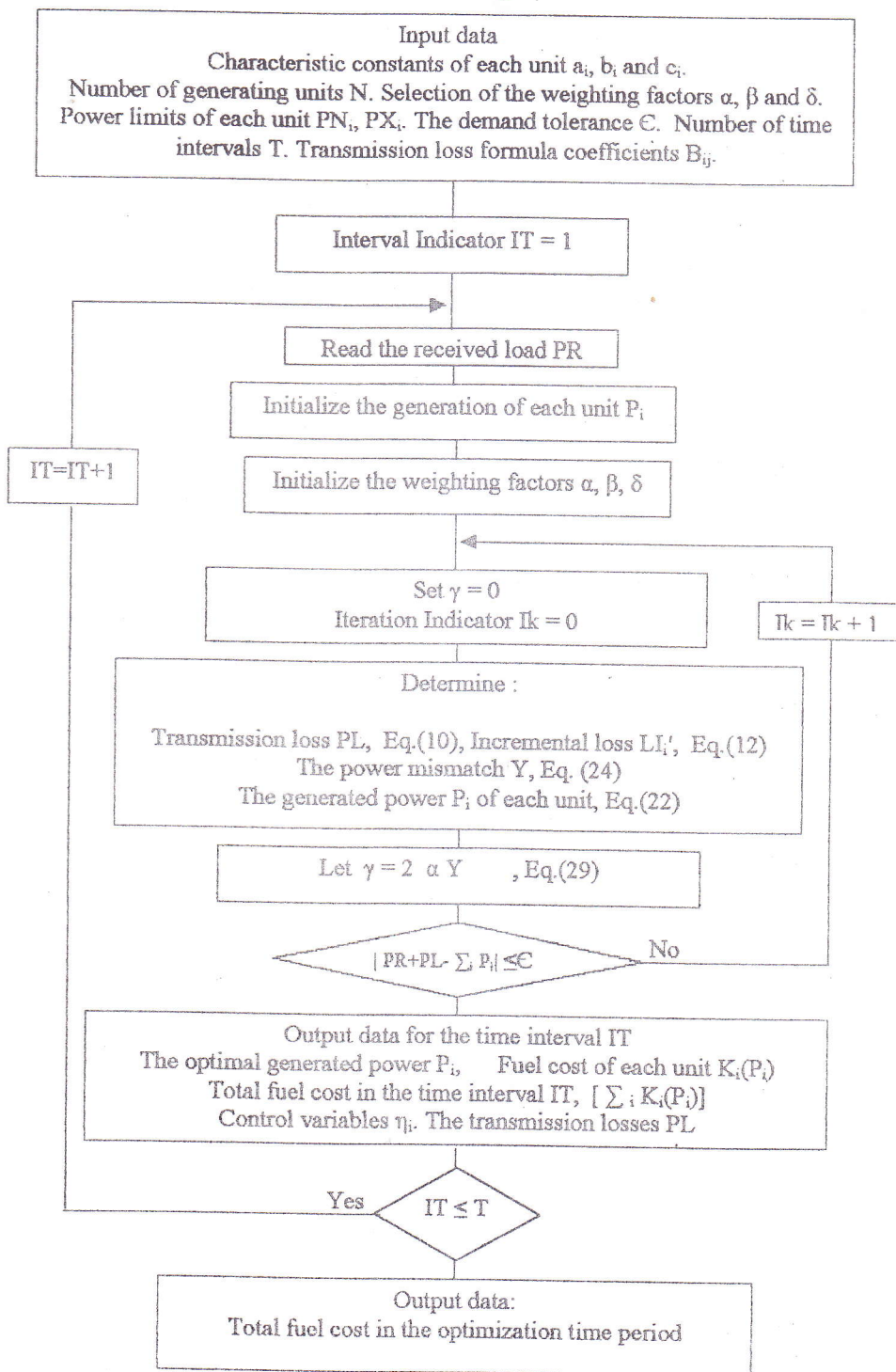


Fig.1 Flow chart of the calculation procedure of the proposed method.



Table 1. Unit fuel cost constants and power limits of each generating unit.

Unit No.	$a_i$	$b_i$	$c_i$	$PX_i$	$PN_i$
1	640.	6.50	0.00180	650.0	90.0
2	500.	7.00	0.00220	600.0	100.
3	296.	7.84	0.00265	500.0	80.0
4	180.	8.20	0.00308	350.0	50.0
5	110.	8.90	0.00340	200.0	30.0

Table 2. Received load (PR) in each time interval (IT).

IT	1	2	3	4	5	6	7	8	9	10	11	12
PR	1500	1240	980	750	570	360	600	830	1025	1150	1300	1490
IT	13	14	15	16	17	18	19	20	21	22	23	24
PR	1620	1400	1250	1000	1280	1450	1680	1900	2150	2300	2010	1800

Table 3. Total fuel cost and the % excess in fuel cost obtained by the used methods when PL aren't considered in the economic dispatch.

Used methods in the economic dispatch		Total fuel cost (£)	% Excess in fuel cost
The proposed method	Without inclusion of power limits constraints	295033.08	0.000745
	With inclusion of power limits constraints	299991.08	1.680565
LaGrange method		295032.86	-
Participation factors method		289281.84	1.949280
Second-order gradient method		297877.39	0.964140

Table 4. Total fuel cost, the % excess in fuel cost and % (TTER) obtained by the used methods when PL are considered in the economic dispatch.

Used methods in the economic dispatch		Total fuel cost (£)	% Excess in fuel cost	Energy Loss (Mwh)	% TTER
The proposed Method	Without inclusion of power limits constraints	302446.44	0.075	848.424	2.68
	With inclusion of power limits constraints	305440.12	1.047	674.211	2.13
LaGrange method		302275.30	-	825.542	2.61

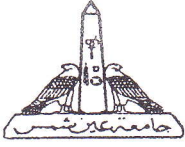
Table 5. Comparison of the test-patterns output data of the artificial neural networks with the obtained output data by LaGrange method and by the proposed method when the transmission losses aren't considered.

PR Mw	P <sub>1</sub>	Without consideration of transmission losses			
		ANN1 learning rate = 0.4  momentum constant = 0.6	LaGrange method	Proposed method	
				Without inclusion of power limits	With inclusion of power limits
1240	P <sub>1</sub>	607.750	601.082	601.082	347.316
	P <sub>2</sub>	364.800	378.158	378.158	330.843
	P <sub>3</sub>	147.000	155.452	155.452	275.050
	P <sub>4</sub>	82.250	75.308	75.308	185.270
	P <sub>5</sub>	26.000	30.000	30.000	101.520
	K	10874.790	11432.910	11432.930	11677.780
1150	P <sub>1</sub>	580.450	571.880	571.880	329.470
	P <sub>2</sub>	328.800	354.260	354.270	312.920
	P <sub>3</sub>	132.500	135.620	135.620	257.050
	P <sub>4</sub>	72.450	58.240	58.240	167.190
	P <sub>5</sub>	22.600	30.000	30.000	83.370
	K	10145.514	10657.890	10657.900	10881.860
1400	P <sub>1</sub>	635.700	650.000	650.000	379.046
	P <sub>2</sub>	433.800	421.814	421.814	362.703
	P <sub>3</sub>	180.000	191.690	191.700	307.060
	P <sub>4</sub>	105.000	106.490	106.490	217.420
	P <sub>5</sub>	35.200	30.000	30.000	133.770
	K	12419.138	12834.120	12834.120	13113.810
2150	P <sub>1</sub>	650.000	650.000	650.000	561.990
	P <sub>2</sub>	597.000	600.000	600.000	546.400
	P <sub>3</sub>	435.500	422.120	422.124	491.610
	P <sub>4</sub>	305.550	304.750	304.750	350.000
	P <sub>5</sub>	159.000	173.130	173.126	200.000
	K	20097.984	19912.870	19912.880	20087.020

Table 6. Comparison of the test-patterns output data of the artificial neural networks with the obtained output data by LaGrange method and by the proposed method when the transmission losses are considered.

PR Mw	P <sub>i</sub>	With consideration of transmission losses			
		ANN2 Learning rate = 0.4 momentum constant = 0.6	LaGrange Method	Proposed method	
				Without Inclusion of power limits	With inclusion of power limits
1240	P <sub>1</sub>	579.800	558.842	558.522	351.747
	P <sub>2</sub>	318.600	352.897	352.518	335.292
	P <sub>3</sub>	192.000	202.473	203.170	279.523
	P <sub>4</sub>	119.350	126.374	127.058	189.760
	P <sub>5</sub>	36.200	30.000	30.000	106.020
	K	11711.320	11716.420	11722.820	11876.680
	PL	30.080	30.595	31.269	22.342
1150	P <sub>1</sub>	551.850	530.595	530.507	333.372
	P <sub>2</sub>	298.200	330.160	329.978	316.842
	P <sub>3</sub>	172.000	180.113	180.470	260.987
	P <sub>4</sub>	102.200	106.037	106.492	171.142
	P <sub>5</sub>	27.800	30.000	30.000	87.341
	K	10788.990	10904.800	10909.690	11055.210
	PL	26.480	26.914	27.447	19.684
1400	P <sub>1</sub>	613.600	601.667	650.000	384.509
	P <sub>2</sub>	355.800	386.436	361.364	368.188
	P <sub>3</sub>	231.000	235.856	229.215	312.571
	P <sub>4</sub>	155.050	157.200	150.705	222.955
	P <sub>5</sub>	57.200	55.473	51.620	139.325
	K	13362.910	13187.880	13238.820	13363.760
	PL	37.140	36.642	42.904	27.546
2150	P <sub>1</sub>	649.350	650.000	650.000	591.125
	P <sub>2</sub>	510.600	589.131	591.711	575.650
	P <sub>3</sub>	415.500	432.224	430.560	500.000
	P <sub>4</sub>	311.850	347.629	345.348	350.000
	P <sub>5</sub>	181.800	200.000	200.000	200.000
	K	19390.170	20621.450	20605.790	20701.640
	PL	65.820	68.994	67.619	66.774

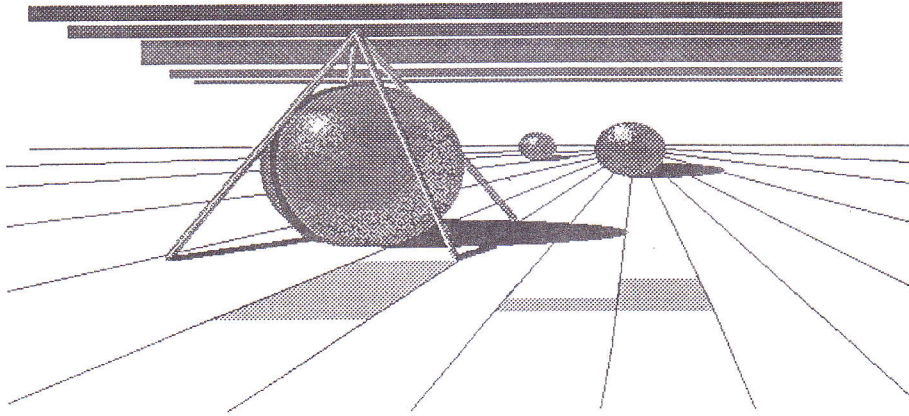




جامعة عين شمس

كلية الهندسة

العدد ٣٧، رقم ٤، ٣١ ديسمبر ٢٠٠٢



النشرة العلمية

الجزء الثالث هندسة ميكانيكية فيزياء ورياضيات